

## Magnetic field generation by ion acoustic waves in an inhomogeneous plasma with temperature gradient

Ghanshyam\*

Department of Physics, B I T, Sindri, Dhanbad-828 123, Bihar, India

and

V K Tripathi

Department of Physics, Indian Institute of Technology, Delhi-110 016, India

E-mail : ghanshyam123@rediffmail.com

*Received 4 October 2002, accepted 25 November 2003*

**Abstract** : An ion acoustic wave propagating perpendicular to the temperature gradient in plasma produces an oscillatory magnetic field via the  $\nabla n \times \nabla T$  mechanism. For a 1 Kev plasma of density  $\sim 10^{21} \text{ cm}^{-3}$ , having temperature scale length  $L_T \sim 35 \mu\text{m}$ , ion acoustic density perturbation  $\sim 3\%$  and wave frequency  $= 10^{11} \text{ rad sec}^{-1}$  One obtains oscillatory magnetic field  $B \sim 30 \text{ KG}$ . This magnetic field is large to influence plasma phenomena.

**Keywords** : Ion acoustic wave, inhomogeneous plasma, temperature gradient, magnetic field.

**PACS Nos.** : 52.38.Hb, 52.20.Hv, 78.20.Ci

Self-generated magnetic fields in laser-produced plasmas have been subject of considerable interest of several years [1–6]. A dominant mechanism for their generation is considered to be the  $\nabla n \times \nabla T$  mechanism : when density and temperature gradients in plasma are not parallel to each other. The electric field produced by the pressure gradient force on electrons possesses a finite curl, generating magnetic field. The temperature gradient in the plasma is produced due to non-uniform heating of electrons by the finite spot size laser whereas the density gradient is caused by the expansion of the plasma away from the target [7–10]. Experiments have reported generation of megagauss magnetic fields that have significant influence on thermal transport [11,12]. Tidmann and Shanny have examined the possibility of excitation of a space periodic magnetic perturbation in plasma where density and temperature gradients are parallel to each other. When a temperature perturbation with  $k$  vector perpendicular to density gradient is present in the system, it produces a space periodic magnetic field through the  $\nabla n \times \nabla T$

mechanism. The magnetic field modifies thermal current density feeding energy to the temperature perturbation. It would be very worthwhile exploring alternate schemes for the generation of spatially and temporally periodic magnetic fields. Currently there is serious interest in such magnetic fields called wigglers for generating high frequency coherent radiation from charged particle beams in a free electron laser [13,14]. The static magnetic wigglers employed in this device have long wiggler wave length  $\lambda_w \geq 2 \text{ cm}$ , hence the operational frequency of FEL,  $\omega_L \approx 4\pi(c/\lambda_w)\gamma_0^2$  (where  $\gamma_0$  is relativistic gamma factor of the beam) is limited. If one could produce a transverse periodic magnetic field of much shorter wavelength (say  $\gamma_w \sim 1 - 10 \mu\text{m}$ ) one could produce high power coherent radiation in the X-ray band.

In this note, we suggest a scheme for the generation of space-time periodic magnetic fields by launching ion acoustic waves in plasma at an angle to density and temperature gradients. The ion acoustic waves could be produced via the stimulated Brillouin and Compton scattering

\*Corresponding Author

processes or the decay instability [15–19]. Since the wavelength of the ion acoustic wave is comparable to the wavelength of the laser, micron size  $B$  fields can thus be produced.

Consider slowly expanding plasma with expansion velocity  $v \ll c_s$ . The plasma has temperature and density gradients,  $\nabla T_0 \parallel \hat{y}$  and  $\nabla n_0 \parallel \hat{z}$ . An ion acoustic wave travels through the plasma in the  $\hat{z}$  direction:  $E = E(y) \exp[-i(\omega t - kz)]$  where  $\nabla \ln n_0 \ll k_{0z}$ . In the adiabatic approximation the temperature and density perturbation due to the sound wave are related to each other as

$$\Delta T = -(T_0/n_0)\Delta n (1 - \gamma), \quad (1)$$

where  $\gamma$  is the ratio of the specific heats at constant pressure and constant volume. In the limit  $\omega \ll k_z v_{th}$  ( $v_{th}$  being the electron thermal speed), one may balance the pressure gradient force with the electric force  $-eE$  on the electrons.

$$\begin{aligned} -eE &= (\gamma/n_0) \nabla n' T_0 - (1/n_0)(n'/n_0) \\ &\nabla(n_0 T_0) - (\gamma/n_0)(n'/n_0)(\nabla n' T_0), \end{aligned} \quad (2)$$

where  $n = n_0 + n' \exp[-i(\omega t - k_z z)]$ .

The magnetic field associated with  $E$  can be obtained from the Maxwell's equation as

$$B = c/(i\omega)(\nabla \times E). \quad (3)$$

Now, we may consider different cases of interest separately.

$n_0$  is uniform,  $T_0$  nonuniform :

In this case,

$$\begin{aligned} B &= \gamma c (\nabla n' \times \nabla T_0)/(i\omega n_0) \\ &= [(\gamma c n')/(i\omega n_0) k_z T_0 / L_T (\hat{z} \times \hat{y})]. \end{aligned} \quad (4)$$

One may take  $n' = n_0 e \phi / T$ , where  $\phi$  is the scalar potential ( $\phi = E_z/(ik_z)$ ) of the ion acoustic wave.

$T_0$  is uniform,  $n_0$  nonuniform :

Here  $B$  turns out to be

$$\begin{aligned} B &= (c/(i\omega e))(T_0/n_0^2) k_z n' \hat{z} \times \nabla n_0 \\ &= (c/(i\omega e))(n'/n_0) k_z (\hat{z} \times \hat{y})(T_0/L_n), \end{aligned} \quad (5)$$

where  $L_n$  is density scale length. Thus, even if a temperature gradient is missing, the density gradient at an angle to the direction of wave propagation may give rise to oscillatory  $B$  field.

$\nabla n_0$  and  $\nabla T_0$  are anti-parallel to each other :

In a laser filament [7,15,16], the electrons are heated to higher temperature via Ohmic heating. This causes redistribution of plasma density via the ambipolar diffusion process [15]. In the steady state one has  $n_0 (T_0 + T_i) \approx$  constant, where  $T_i$  is the ion temperature and  $T_0 \sim m_i v_0^2/3$ ,  $m$  is the ion mass and  $v_0$  is the oscillatory electron velocity due to the laser.

When an ion acoustic wave is generated in the filament by the SBS [16–18] process, one obtains

$$B = ((\gamma - 1)/(\omega e))(n'/n_0) c k_z (\hat{z} \times \hat{y}) T_0 / L_T, \quad (6)$$

For the following set of parameters :

$T_0 = 1$  Kev,  $c = 3 \times 10^{10}$  cm s<sup>-1</sup>,  $c_s = 3.09 \times 10^7$  cm sec<sup>-1</sup>,  $n'/n_0 = 0.03$ ,  $L_T = L_N = 35 \times 10^{-4}$  cm,  $n_c$  (critical density of plasma) =  $10^{21}$  cm<sup>-3</sup>,  $\gamma = 1.4$ ,  $\nu_{ei}$  (electron ion collision frequency) =  $1.3 \times 10^{10}$  sec<sup>-1</sup>,  $\lambda_m$  (mean free path) = 0.13 cm.

For the present calculation,  $\omega_c$  (electron cyclotron frequency) =  $4.8 \times 10^{11}$ ,  $7.2 \times 10^{11}$ , and  $2.4 \times 10^{11}$  rad s<sup>-1</sup> are obtained, corresponding to periodic  $B$  fields of 30, 45 and 15 K Gauss respectively.

The magnetic field affects thermal transport when  $\omega_c \tau > 1$  (where  $\tau = 1/\nu$  is the collision time) and the electron larmor radius is smaller than the scale lengths of magnetic field, density and temperature variations. It has significant effect on Ohmic energy dissipation and temperature profile in the underdense region. The ratio of energy loss via thermal conduction to collisional loss is given by

$$R = m/M [(width \text{ of the beam})/(mean \text{ free path})]^2 (\nu^2 + \omega_c^2)/\nu^2,$$

where  $m$  and  $M$  are mass of electron and ion.  $R$  is strongly affected by  $B$  field for  $\omega_c > \nu$ . For a 1 Kev plasma of density  $\sim 10^{21}$  cm<sup>-3</sup>,  $L_T \approx 35 \mu\text{m}$ , density perturbation  $\approx 3\%$ , one obtains  $\omega_c \tau$  to be  $\sim 1$ . At lower densities,  $\nu$  is smaller hence  $\omega_c > \nu$ .

Self-generated magnetic field  $B$  scales as  $\omega^{-1}$ , hence for langmuir wave, it is not important.

The transverse periodic magnetic fields of wavelengths 1  $\mu\text{m}$  thus generated, can be used as wigglers to produce high power coherent radiation in the X-ray band.

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